On the use of the log-normal particle size distribution to characterize global rain

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I. INTRODUCTION AND BACKGROUND

Radar and microwave radiometer measurements from low-earth orbit provide a means to characterize rainfall over much of the globe. In the case of the TRMM Precipitation Radar, monthly accumulations of data in $5^0 \times 5^0$ and $0.5^0 \times 0.5^0$ degree boxes serve as the basis for calculating histograms and first-order statistics of rain rate and reflectivity factor at several heights. Apart from calculations of diurnal variations of rainfall and important studies on the Madden-Julian oscillation and ENSO, spatial and temporal correlations of rainfall at smaller space-time scales have received less attention.   Nevertheless, these second-order statistics are important for a number of studies including algorithm testing and development, partial-beam filling studies, satellite sampling studies, and fade statistics along satellite communication links.   Perhaps most importantly, the measurements may provide information from which to construct a model of global rainfall.   With advances in ground-based polarimetric and air- and satellite-based dual-wavelength radars, which offer the potential of providing characteristics of the particle size distribution, continued investigation of space-time models of both rainfall and drop size distribution (DSD) is warranted. Particular questions of interest are: how can such models be constructed, what parameters are required for their specification, and how well do various measurements and instruments provide such information. More generally, can such models, derived from radar data, provide a means by which to characterize rainfall and size distributions over a range of space-time scales.

In this paper we focus on a log-normal model for the raindrop size distribution. By assuming that the rain rate and associated variables are log-normally distributed, it follows that the parameters of the DSD can be modeled as random variables from a multivariate Gaussian distribution. We go on to discuss in a preliminary fashion how the rain rate and DSD parameters can be generated by simulating the underlying Gaussian process.

II. LOG-NORMAL DROP SIZE DISTRIBUTION

The log-normal approximation for the raindrop diameter distribution is [1]:

$$N(D) = N_t \exp[- (\log D - \eta)^2/2\sigma^2] / [(2\pi)^{0.5} D] \qquad (1)$$

where N_t is the particle number concentration (m^{-3}) and D is the diameter (mm).

The natural logarithm of the pth moment, M_p, of N(D) is

$$\log(M_p) = \log(N_t) + p\eta + (p\sigma)^2/2, \qquad (2)$$

which can be expressed in summation or vector notation as

$$\log(M_p) = \Sigma\, c_i\, U_i = \mathbf{c}(p)\cdot\mathbf{U} \qquad (3)$$

where

$$\mathbf{U} = [\log(N_t), \eta, \sigma^2], \quad \mathbf{c}(p) = [1, p, p^2/2] \qquad (4)$$

so that the mean of $\log(M_p)$ and the covariance of the logarithms of the pth and qth moments can be written, respectively, as

$$\langle\log(M_p)\rangle = \Sigma\, c_i(p)\, \langle U_i\rangle = \mathbf{c}(p)\cdot\langle\mathbf{U}\rangle \qquad (5)$$

$$A(p, q) \equiv \mathrm{cov}(\log M_p, \log M_q) = \Sigma_{i,j}\, c_i(p)\, c_j(q)\, \mathrm{cov}(U_i, U_j) \quad (6)$$

We note as an aside that (5) and (6) can be used to derive parameters in the power-law relations between radar and meteorological quantities in terms of the DSD parameters. They also can be used to address the TRMM-related problem of expressing 'a' in the R=a Z^b relationship in terms of α and β in the k = αZ^β relationship.

III. GENERATION OF RAIN AND DSD FIELDS

One of the motivations for considering the log-normal DSD is that the logarithm of any moment can be expressed as a linear combination of the 3 parameters of the distribution, $[\log(N_t), \eta, \sigma^2]$. If the rain rate, R, is assumed to be log-normal and if power-laws between R and Z (reflectivity factor), and R and k (specific attenuation) are valid, then (2) holds for at least 3 different moments of the DSD. In general, we can invert the 3 equations and solve for the U_i (i=1,2,3) in

terms of log R, log Z, and log k. But since U_i can be written as a linear combination of Gaussian variables, the U_i themselves are Gaussian. It is also worth noting that the mass-weighted diameter, D_m, and the median mass diameter, D_0, can be expressed as $M_4/M_3 = \exp(\eta + 7\sigma^2/2)$ and $\exp(\eta + 3\sigma^2)$, respectively, so that log D_m and log D_0 are also Gaussian. Although there is some controversy regarding the log-normality of R, Z, and k, over large space-time regions the assumption appears to be a reasonable and useful approximation for a number of studies. That **U** is Gaussian permits us to construct realizations of the size distribution in space from which we can deduce all the usual bulk radar and meteorological parameters. It also provides a means of computing radar and radiometric fields directly from such fields.

A number of approaches to generate Gaussian random fields are described in the literature. The single-variate, multidimensional form of the equation considered here was given by Shinozuka and Jan [2]

$$f(x,y) = \sum_{i,j} [S(k_{1i}, k_{2j}) \Delta k_1 \Delta k_2]^{0.5} \cos(k'_{1i} x + k'_{2j} y + \phi_{i,j,}) \quad (7)$$

where S is the 2D Fourier transform of the autocovariance of f(x, y), $\phi_{i,j}$ are independent random phases uniformly distributed over $[0, 2\pi]$ and the k_{1i} are given by $k_{1i} = k_{1min} + (i-0.5)\Delta k_1$, i=1,..,N with $\Delta k_1 = (k_{1max} - k_{1min})/N$ where the 2-sided spectrum S is assumed to be negligible outside the region between k_{1min} and k_{1max}. Finally, $k'_{1i} = k_{1i} + \delta k_i$, where δk_i is a uniformly distributed random variable with a maximum value much less than Δk_1. Similar comments apply to the k_{2j} variables. The multivariate form of (7) can be written in a similar form [2]; however, in this case, the Fourier transforms of all autocovariances and cross covariances are required. For the 3-parameter log-normal distribution, this requires three autocovariances, $\log(N_t)$, η, σ^2, and three cross-covariances. For example, the cross-covariance of $(\log(N_t), \eta) = (U_1, U_2)$ can be written:

$$E[(U_1(\underline{x}_1, t_1) - E(U_1))(U_2(\underline{x}_2, t_2) - E(U_2))] = C_{U1,U2} (\rho, \tau) \quad (8)$$

where E represents the expectation operator. Because the fields are assumed to be isotropic and homogeneous in space and stationary in time, the cross-correlations and autocorrelations are functions only of the distance between points in space, ρ, and the absolute difference in time, $\tau = |t_1 - t_2|$. For the remainder of the paper we will consider experimental data and some of the measurement and modeling efforts needed to specify the covariance functions.

The left-hand images of Fig. 1 show the estimated rain rate within the swath of the TRMM radar for two satellite passes over a $10^0 \times 10^0$ latitude-longitude box that includes the Melbourne, FL WSR-88D ground-based radar. On the right-hand side are shown the autocorrelations of the rain rate field.

Fig. 2 shows the spatial correlation versus distance from an average of the individual correlation overpass data acquired during Aug. 1998. In this example, 15 overpasses were acquired with rain exceeding 10% of the total area and the number of PR (precipitation radar) scans within the box exceeding 100, comprising more than 400 km of along-track data.
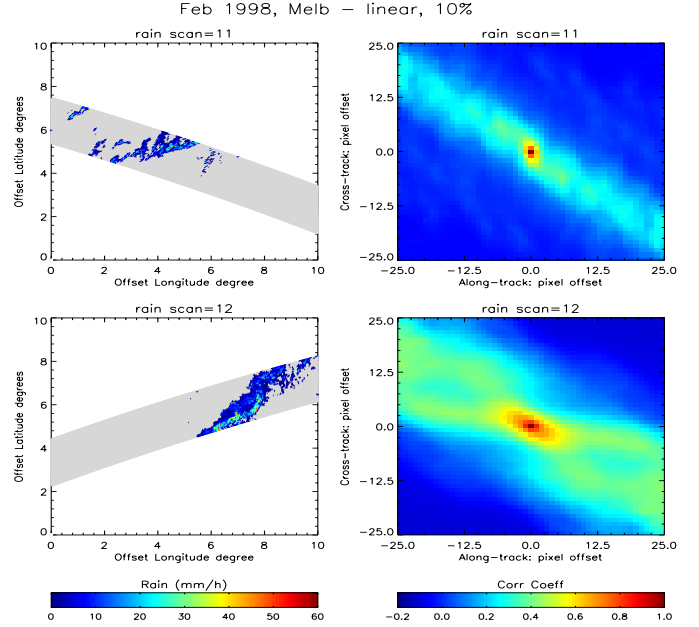

Fig. 1: Two TRMM PR overpasses of the Melbourne, FL site in Feb. 1998 (left) and the corresponding spatial correlations.
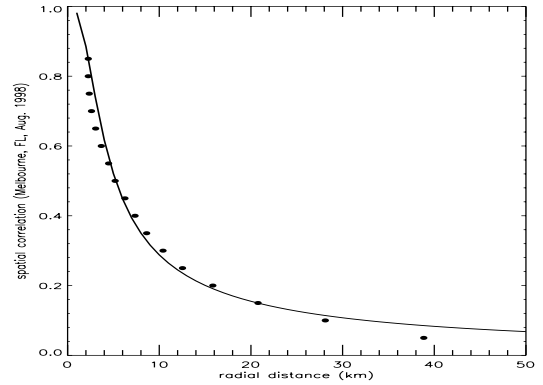

Fig. 2: Average spatial correlation of rain rate from individual overpasses of the Melbourne FL site for Aug. 1998. Circles correspond to the experimental data; the solid line to a fit.

In this case, the normalized spatial correlation can be modeled reasonably well by:

$$C(\rho) = a^{2\nu} /(\rho^3 + a^2)^\nu$$

where for the solid curve in Fig. 2, a = 2, ν = 0.3. TRMM Precipitation Radar (PR) data from eight regions of the globe

are presently being used to estimate the spatial correlation of the rain rate as functions of season and rain type. However, the temporal sampling of the TRMM satellite is much too coarse to estimate the temporal correlation of the rain and ground-based data are needed.

Following Bell [3] we can construct realizations of the rain field from the 2-dimensional Gaussian process generated by (7). Two such simulations are shown in Fig. 3. The simulated rain fields reproduce fairly well the fractional rain coverage, mean, standard deviation and spatial correlation of the inputs. It should be noted, however, that these results are preliminary and that further testing and development of the model is required.
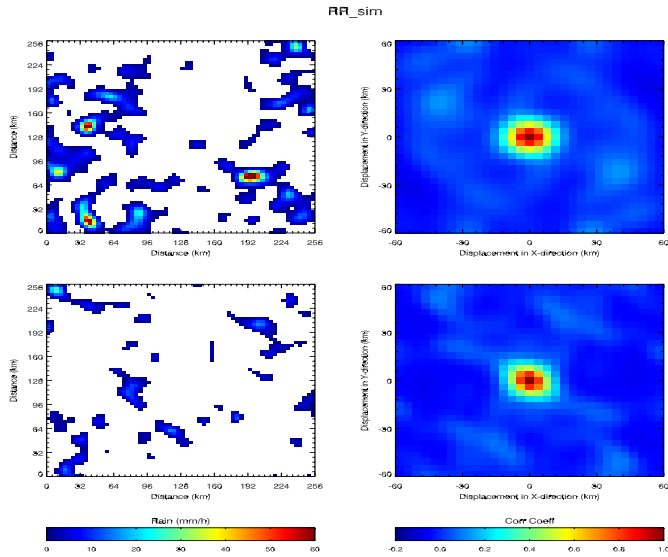

Fig. 3: Two realizations of a rain field with spatial correlation based on that shown in Fig. 2.

As noted earlier, generating realizations of the three DSD parameters requires data in space and time to model the six covariances. Less complete but still quite useful would be a two parameter DSD consisting of a particle concentration and a characteristic size. In this case, the number of covariances needed would be three. Ground-based polarimetric radar data are well suited for this task but are limited in space by the relatively small number of well-calibrated radars of this type. Spaceborne dual-wavelength radar, as proposed in the Global Precipitation Mission (GPM), would provide extensive spatial coverage but with little or no information on the temporal evolution of the size distribution. Airborne dual-wavelength radar and ground-based disdrometer data, while limited in many respects, should prove useful in preliminary studies. Shown in Fig. 4 are DSD parameters derived from disdrometer data averaged along a 2.3 km path, measured at Wallops Island, VA. Corresponding temporal autocorrelations of the parameters are shown in the lower set of panels. Size distribution data derived from airborne radars
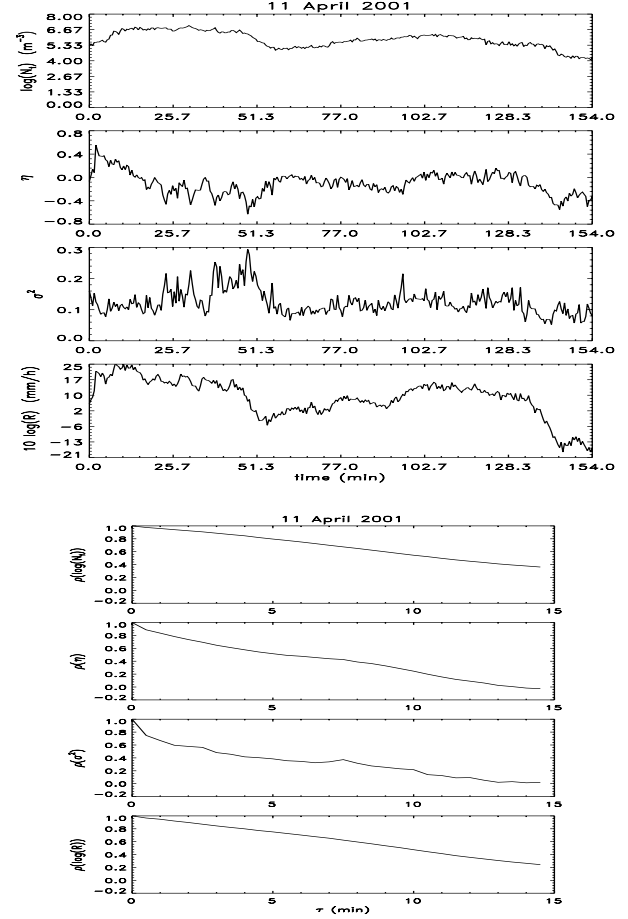

Fig. 4: Estimated log-normal parameters from disdrometer data over 2.3 km path (top set) along with temporal autocorrelations of the data (bottom).

provide similar information for the spatial characterization.

IV. Summary and Conclusions

Parameters in the log-normal DSD can be approximated as jointly Gaussian. This fact can be used to simplify the generation of DSD fields and may prove useful in comparing retrievals from ground-based and spaceborne radar data. The model may also find applications in generating realistic radar and radiometric fields for algorithm testing and development.

References

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